Lebanese University
Faculty of Sciences 1

I2205 E – INFO318 E

Graph Theory

Partial 2019-2020
Tue. Dec. 13th, pm
Duration: 1 hour

Exercise I: Fundamentals (7 points)

Part A

Consider the directed graph $G = (V, E)$ where $V = \{a, b, c, d, e\}$ and $E = \{(a, b),\ (a, c),\ (b, c),\ (b, d),\ (c, d),\ (c, e),\ (d, e)\}$.

1. Find the adjacency matrix **and** the adjacency list of the graph G. Deduce the out-degrees and the in-degrees of the vertices.
2. Is this graph bipartite? Why or why not?
3. Is this graph Eulerian? Deduce a necessary condition for a directed graph to be Eulerian.

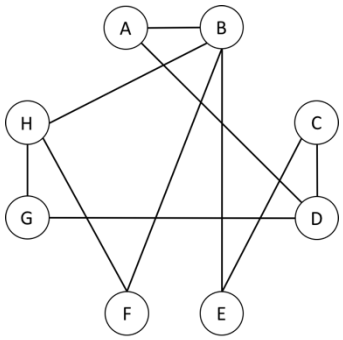
Part B

Four couples met in a conference, where every one has to shakehand with the others, except for his spouse.

1. Represent the given problem by a graph where the vertices and the edges be determined.
2. Is the obtained graph complete? Eulerian? Explain.
3. Is the obtained graph Hamiltonian? If yes give a Hamiltonian cycle. What does such a cycle mean?

Exercise II: Graph coloring (6 points)

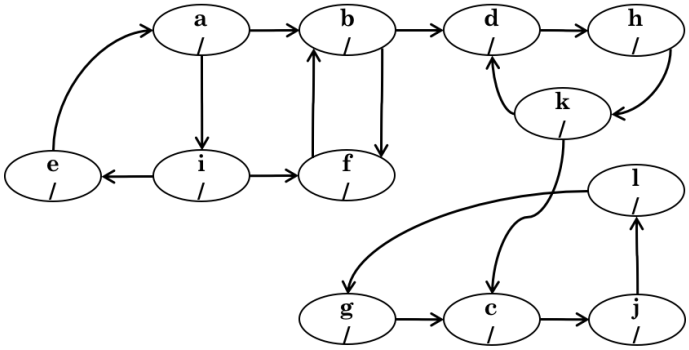
Consider the following graph $G = (V, E)$:



1. Maximize and minimize the chromatic number, χ(G), of G.
2. Applying the algorithm of your choice, propose a good coloring for the vertices of G.
3. Propose a good coloring for the edges of G.

Exercise III: Graph Traversal (7 points)

Consider the following graph $G = (V, E)$:



1. Apply the DFS algorithm on the given graph starting at vertex a. If there is a decision to be taken between several vertices, choose the letter closest to the beginning of the alphabet first. Indicate the discovering time (d), finishing time (f), the tree edge (T), forward edge (F), backward edge (B), and the cross edge (C).
2. Give the DFS tree.
3. Does this graph have bridges? If yes, give them.
4. Determine the circuits in G and give the reduced graph G_R.